INTERACTION OF ACOUSTIC WAVES WITH BURNING SURFACES OF CONDENSED SYSTEMS

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The principal topic of the linear theory of the acoustic burning stability of solid propellant systems is the possibility of amplifying acoustic pressure waves reflected from a burning surface. The investigation of reflected acoustic waves reduces to the study of the reorganization of the physico-chemical processes in the combustion zone, caused by harmonic pressure disturbances, and to the calculation of the acoustic perturbation of the rate at which the gas flows off the burning surface. The reflectivity of the combustion zone is characterized by the value of the acoustic admittance.

It is not possible to obtain an exact solution of the problem of the acoustic admittance of a burning surface, because the complex processes taking place in the combustion zone of a solid propellant are not amenable to rigorous analytical description, even in the stationary case. Analysis is based on simplified models of the combustion zone which take into account only the most significant peculiarities of the combustion mechanism of solid propellants [1–7].

The problem of acoustic admittance of a burning surface has been investigated in detail in [1, 2, 6]. In [1, 2], the assumptions concerning the combustion zone include the hypothesis of constant combustion temperature in steady-state conditions. It may be shown that this is possible only when the value of the total heat release in the combustion zone does not remain constant in steady-state conditions and, at certain moments of time, exceeds the initial enthalpy of the solid propellant, i.e., the magnitude of the thermal effect of the combustion reaction. Such an assumption can be hardly justified from the physical point of view.

In [5], the acoustic admittance of a burning surface was calculated on the basis of Zel'dovich's theory of solid-propellant combustion under variable pressure [8].

This paper did not take into account the exothermic chemical reaction in the condensed phase (k-phase) and its effect on the burning rate, nor did it allow for the change in surface temperature in the k-phase in unsteady conditions.

The aim of the present paper is to obtain formulas for the acoustic admittance of a burning surface with allowance for these factors. As distinct from [1, 2], we assume as in [5] that the total heat release rather than the combustion temperature is constant in unsteady conditions. This, and some other differences in the steady-state combustion model, leads to results that differ from the conclusions of [1, 2].

The problem is solved on the assumption that the period of the acoustic oscillations is large compared to the characteristic time of the processes that take place in the gas and in the chemical reaction zone of the k-phase (quasi-stationary approximation). Such an approach is justified for frequencies not higher than the order of 10^4 cps. This frequency range includes essentially the frequencies characteristic of the vibrational modes of powder burning.

In order to investigate the reflection of acoustic waves of a frequency higher than 10^4 cps from a burning surface it is necessary to take into account the inertial properties of the processes occurring in the gas and the reaction zones.

In the frequency range studied, the dimensions of the combustion zone in the gas are much smaller than the acoustic wavelength, and therefore it is safe to assume that the pressure fluctuations within the limits of the combustion zone are independent of the space coordinate.

§1. Combustion-zone model. The combustion zone of a solid propellant may be approximated as consisting of several regions: (1) the thermal layer in the k-phase, in which no chemical reactions take place, (2) a zone with chemical reactions in the k-phase, adjacent to the phase interface, (3) a heated region in the gas, free of chemical reactions, (4) a reaction zone in the gas, and (5) gaseous combustion products. A graphical representation of the combustion zone is given in Fig. 1.

For most solid-propellant systems, the characteristic durations of processes in regions (1) through (4) are as follows [1]: $\tau_1 \approx 0.3 \cdot 10^{-3}$ sec, $\tau_2 \approx 7 \cdot 10^{-6}$ sec, $\tau_3 \approx 2 \cdot 10^{-5}$ sec, and $\tau_4 \approx 10^{-6}$ sec.

The slowest are the processes in the thermal layer of the k-phase. At frequencies up to $n \approx 10^4$ cps, the unsteady processes in zones (2) through (4) may be regarded as depending parametrically on time through the boundary conditions.

It is postulated that the temperature dependences of the chemical reaction rates $\Phi_1(T)$ and $\Phi_2(T)$ in the k-phase and the gas are such that the reactions occur within narrow temperature ranges, close to the surface temperature T_S and combustion temperature T_2. This assumption holds for sufficiently large values of the activation energies in the k-phase E_S and the gas E_2; $E_S \gg RT_S$, $E_2 \gg RT_2$, where R is the gas constant. It is assumed that the values of the heat release, i.e., the completeness of a chemical reaction, in the reactive zones do not change under the effect of the acoustic field, remaining equal to Q_1 and Q_2, respectively.

We shall now derive the equations that relate the linear disturbances of the parameters in regions (1) through (4) in the case of harmonic pressure variations.

§2. Heating zone in the k-phase. The thermal processes in the heating zone of the k-phase are described by the unsteady heat conduction equation without heat sources. The steady temperature distribution is the so-called Michelson profile

$$T(x) = T_0 + (T_s - T_0)\exp\left(\frac{U}{\varkappa}x\right), \qquad (2.1)$$

where T_0 is the initial temperature of the k-phase, U is the linear burning rate, $\varkappa$ is the thermal diffusivity, T_S is the surface temperature of the k-phase. The extent of the reaction zone in the k-phase and the temperature difference at its boundaries are small, so that in the derivation of the equations for the thermal layer of the k-phase it may be assumed that the region (2) is a surface that coincides with that of the solid propellant (x = 0).

If the pressure and burning rate experience small harmonic disturbances of frequency ω, then the temperature distribution in the k-phase differs from the steady-state profile (2.1) by a small value $\delta T(x) \cdot \exp(i\omega t)$.

The equation for the amplitude of the temperature disturbance has the form

$$\varkappa \frac{d^2\delta T}{dx^2} - U\frac{d\delta T}{dx} - i\omega\delta T = = \frac{U}{\varkappa}(T_s - T_0)\,\delta U \exp\left(\frac{U}{\varkappa}x\right), \qquad (2.2)$$

where δU is the amplitude of the burning-rate disturbance.

The solution of Eq. (2.2) that satisfies the boundary conditions $\delta T(-\infty) = 0$, $\delta T(0) = \delta T_s$, where T_s is the amplitude of the disturbance of k-phase surface temperature, has the form

$$\delta T(x) = \left(\delta T_s - i\frac{\varphi}{\omega}\delta U\right)\exp\left(\frac{U}{2}\frac{\beta_1}{\varkappa}x\right) + $$

$$+ i\frac{\varphi}{\omega}\delta U \exp\left(\frac{U}{\varkappa}x\right), \qquad \beta_{1,2} = 1 \pm \sqrt{1 + 4i\Omega},$$

$$\Omega = \frac{\varkappa\omega}{U^2}, \qquad \varphi = \frac{U}{\varkappa}(T_s - T_0). \qquad (2.3)$$

Differentiating (2.3) on the assumption that $x = 0$, we obtain a relation between the amplitudes of the temperature disturbances δT_s, the temperature gradient $\delta\varphi$, and the mass velocity of combustion δm at the "hot" boundary of the thermal layer in nonsteady-state conditions:

$$\frac{\delta\varphi}{\varphi} = \frac{\beta_1}{2}\frac{\delta T_s}{T_s - T_0} + \frac{i\beta_2}{2\Omega}\frac{\delta m}{m}, \qquad (2.4)$$

where $m = \rho_1 U$, $\delta m = \rho_1\delta U$, $\rho_1 = \text{const}$ is the density of the k-phase.

§3. **Reaction zone in the k-phase.** Let us assume that a zero-order chemical reaction takes place near the surface of the k-phase. In quasi-stationary approximation, the thermal processes in the reaction zone are described by the steady-state heat conduction equation. To obtain a relation between the disturbances in this zone, we shall use the relationships for the steady-state propagation rate of the front of an exothermic chemical reaction in the k-phase [9].

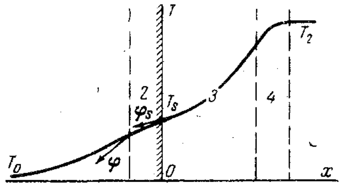

Fig. 1

The equations for the propagation rate of the reaction front may be written in the approximate form

$$\lambda^2\varphi^2 - \lambda^2\varphi_s^2 = 2\lambda Q_1\rho_1\int_{T_0}^{T_s}\Phi_1(T)\,dT,$$

$$\lambda\varphi - \lambda\varphi_s = mQ_1, \qquad (3.1)$$

where φ, φ_s are the temperature gradients at the boundaries of the reaction zone, λ is the heat conductivity, and m is the mass velocity of gasification. After linearization, we obtain from (3.1) an equation that relates the velocity disturbances with the temperature disturbances and the temperature gradient at the surface of the k-phase:

$$(1 - \mu)\frac{\delta m}{m} - z_1\frac{\delta T_s}{T_s - T_0} + \frac{\delta\varphi}{\varphi} = 0, \qquad \mu = \frac{Q_1}{c_1(T_s - T_0)},$$

$$z_1 = \frac{\lambda\Phi_1(T_s)}{m^2 c_1}, \qquad (3.2)$$

where c_1 is the heat capacity of the k-phase.

§4. **Thermal layer in the gas.** In the quasi-stationary approximation employed, the energy balance equation in the region (2) through (4) has the form

$$m(c_1 T_s + Q_1 + Q_2) - \lambda\varphi = mc_2 T_2, \qquad (4.1)$$

Table 1

	n	0	10	50	100	200	300	500	800	1000	2000	5000	7000	10000
1	R_1	500	498	471	421	328	249	119	—33	—118	—439	—1000	—1242	—1514
	R	500	498	469	416	319	244	153	115	130	346	793	928	1045
2	R_1	500	498	471	423	328	249	119	—33	—118	—439	—1000	—1242	—1514
	R	500	498	471	420	325	246	121	—13	—077	—219	25	280	606
3	R_1	500	498	471	428	328	249	196	—33	—118	—439	—1000	—1242	—1514
	R	500	499	471	422	329	251	126	—14	—86	—308	—361	210	083
4	R_1	500	499	473	427	346	281	184	82	30	—137	—349	—420	—488
	R	500	499	474	431	356	299	219	145	113	40	77	149	271
5	R_1	500	499	472	422	327	246	112	—51	—145	—514	—1256	—1626	—2095
	R	500	498	470	417	313	222	68	—111	—203	—407	77	444	815
6	R_1	500	499	474	428	347	283	196	130	118	317	1843	2724	3548
	R	500	498	471	413	297	185	—32	—351	—551	—1010	444	863	1129
7	R_1	300	298	260	190	59	—51	—233	—446	—566	—1015	—1800	—2139	—2520
	R	300	298	259	189	56	—55	—230	—419	—508	—707	—365	—7	448
8	R_1	700	699	683	653	597	549	471	380	329	136	—200	—345	—508
	R	700	699	683	652	595	547	473	392	353	268	415	568	768
9	R_1	500	499	473	425	336	261	138	—8	—89	—394	—929	—1159	—1419
	R	500	499	473	426	337	263	144	12	—55	—230	79	529	1156
10	R_1	500	499	470	418	320	237	101	—59	—148	—484	—1072	—1325	—1610
	R	500	499	470	427	317	234	103	—37	—105	—254	2	270	610
11	R_1	500	498	469	414	312	225	83	—83	—177	—529	—1143	—1408	—1706
	R	500	498	468	413	309	222	85	—62	—132	—288	—201	260	615

where c_2 is the heat capacity of the gas, and T_2 is the flame temperature.

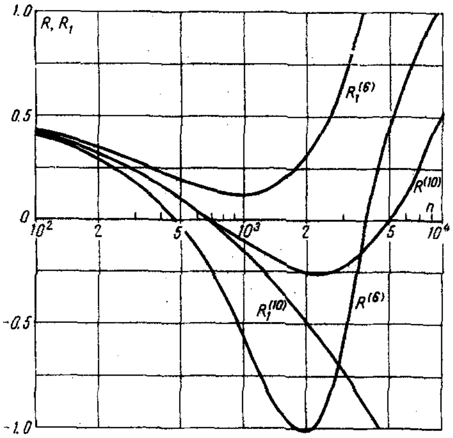

Fig. 2

Linearization of this equation yields

$$\frac{\delta m}{m}+\frac{\delta T_s}{T_s-T_0}-\frac{1}{\tau}\frac{\delta T_2}{T_2}-\frac{\delta\varphi}{\varphi}=0\quad \tau=\frac{c_1(T_s-T_0)}{c_2T_s}. \tag{4.2}$$

§5. **Chemical reaction zone in the gas.** The relations between the disturbances in zone (4) are obtained on the assumption that we know the steady-state dependence of the mass velocity of flame front propagation in the gas $m = m(p, T_2)$. This can be determined experimentally or theoretically, e.g., from the Zel'dovich—Frank-Kamenetskii formula.

For small disturbances, we have

$$\frac{\delta m}{m}=\nu\frac{\delta p}{p}+\frac{\varepsilon}{\tau}\frac{\delta T_2}{T_2},\qquad \varepsilon=\left(\frac{\partial\ln U}{\partial T_0}\right)_p(T_s-T_0),$$
$$\nu=\left(\frac{\partial\ln U}{\partial\ln p}\right)_{T_0}. \tag{5.1}$$

§6. **Acoustic admittance.** In the frequency range studied, the acoustic wavelength is much larger than the width of the combustion zone, so that in the investigation of the acoustic properties of a burning surface in the k-phase, the combustion zone may be regarded as both infinitely thin and coinciding with the k-phase surface, and its acoustic properties may be characterized by the acoustic admittance

$$\zeta=-\rho c\frac{\delta u}{\delta p}, \tag{6.1}$$

where δp is the acoustic pressure in the combustion zone, c is the speed of sound, ρ is the gas density, and u is the acoustic disturbance of the gas velocity at the "hot" boundary of the combustion zone.

In order to obtain the function $\zeta(\omega)$ in explicit form, it is necessary to establish a relation between the quantities δu and δp with the aid of the equations that describe the rearrangement of the combustion zone under the influence of harmonic pressure variations.

It should be taken into account that the interaction of an acoustic wave with a burning surface leads to the onset of density and temperature disturbances that cannot be carried off by the reflected acoustic wave alone. The reflection of an acoustic wave is accompanied by the formation of an entropy wave that propagates at the same rate as the gas flow [7, 10]. According to [5], the formula relating an acoustic disturbance of the gas velocity with the disturbances of the thermodynamic variables at the "hot" boundary of the zone in the presence of an entropy wave has the form

$$\frac{\delta u}{u}-\frac{\delta m}{m}+\frac{\delta p}{p}-\frac{\delta T_2}{T_2}=0. \tag{6.2}$$

Making use of the equations (2.4), (3.1), and (3.2), we shall express the quantities δm and δT_2 in terms of δp. Substituting the relations obtained into Eq. (6.2), we get a relation between δu and δp.

Further, after substitution of the dependence of δm on δp into the formula (6.1), we obtain an expression for the acoustic admittance of the burning surface:

$$\zeta=\frac{u}{\gamma c}\Big\{1-\nu+\nu(\tau+\varepsilon)\times$$
$$\times\frac{1-Z+(2-\beta_1)(2-\mu)[2(z_1-1)]^{-1}}{(1-Z)\varepsilon-1+(2-\beta_1)(2\varepsilon-\varepsilon\mu-1)[2(z_1-1)]^{-1}}\Big\},$$
$$Z=\frac{\sqrt{1+4i\Omega}}{2i\Omega},\qquad \gamma=\frac{c_p}{c_v}, \tag{6.3}$$

where c_p, c_v are the specific heats of gaseous combustion products at constant pressure and constant temperature, respectively. The onset of acoustic instability in the combustion of solid propellants is associated with the possibility that the acoustic waves are amplified on reflection from the burning surface. A criterion for amplification is the sign of the real part of the acoustic admittance, which may be written in the form

$$\mathrm{Re}\left(\frac{c}{\gamma u}\zeta\right)=R=\frac{-f_1(x,\varepsilon,\nu,\tau)+2Bbx(x^2-1)(x-x_1)}{f_2(x,\varepsilon)+2b^2x(x^2-1)(x-x_2)}, \tag{6.4}$$

$$x_1=\Big\{(z_1-1)[(2-\mu)(2\varepsilon-\nu\varepsilon+\nu\tau)-$$
$$-2(2-\mu)\varepsilon(\varepsilon+\nu\tau)-(1-\nu)(2-\varepsilon)+\varepsilon+\nu\tau]\Big\}\times$$
$$\times\Big\{[(2-\mu)(\varepsilon+\nu\tau)-(1-\nu)][1-\varepsilon(2-\mu)]\Big\}^{-1},$$
$$x_2=\frac{2(\varepsilon-1)(z_1-1)}{(2-\mu)\varepsilon-1},\qquad x=\left[\frac{1}{2}+\frac{1}{2}(1+16\Omega^2)^{1/2}\right]^{1/2},$$
$$f_1(x)=a_1x^2+b_1x+c_1,$$
$$a_1=(1-\varepsilon)[\nu(1+\tau)-(1-\varepsilon)],$$
$$b_1=\nu(1+\varepsilon\tau)-(1-\varepsilon^2),\qquad c_1=\varepsilon(\nu-2)-\nu\tau,$$
$$f_2(x)=(1-\varepsilon)^2x^2+(1-\varepsilon^2)x+2\varepsilon,$$
$$B=\frac{(2-\mu)(\varepsilon+\nu\tau)-(1-\nu)}{2(z_1-1)},\qquad b=\frac{\varepsilon(2-\mu)-1}{2(z_1-1)}.$$

Amplification of acoustic waves on reflection from the burning surface of a solid propellant occurs if

$$\mathrm{Re}\left(\frac{c}{\gamma u}\zeta\right)<0.$$

Table 2

	1	2	3	4	5	6	7	8	9	10	11
μ	−0.5	0.5	0.7	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
z_1	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0	10.0
τ	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.10	0.20	0.25
ε	0.9	0.9	0.9	0.7	1.0	1.2	0.9	0.9	0.9	0.9	0.9
ν	0.5	0.5	0.5	0.5	0.5	0.5	0.7	0.3	0.5	0.5	0.5

The frequency range in which the burning surface amplifies the acoustic waves may be determined by substituting specific values of the parameters ε, ν, τ, μ, and z_1 into the formula (6.4) with subsequent analysis of the sign of R for various values of x and, hence, of the frequency of the oscillations.

It should be noted that the formula for the acoustic admittance of a burning surface derived in [5] is contained in (6.4) as a special case when $z_1 \to \infty$. The corresponding expression for the real part of the acoustic admittance has the form

$$\operatorname{Re}\left(\frac{c^{-}}{\gamma u}\zeta\right)_{z_1=\infty} = R_1 = -\frac{f_1(x,\varepsilon,\nu,\tau)}{f_2(x,\varepsilon)}. \qquad (6.5)$$

Values of R_1 and R computed for various values of the frequency n in the range $0 \ll n \ll 10^4$ cps are given in Table 1. The values used for the parameters ε, ν, τ, μ, and z_1 are given in Table 2. In one case. the calculations are performed for $\mu < 0$. Curves showing the dependence of the real part of the acoustic admittance on the frequency in the cases 6 and 10 are given in Fig. 2.

From Table 2 it can be seen that the tendency of a propellant toward acoustic combustion instability increases with increasing ε, τ, and ν.

Furthermore, it may be seen that heat release in the k-phase stabilizes the combustion process in some cases, and may have the contrary effect in other cases ($\varepsilon > 1$).

The theoretical data obtained are found to be in good qualitative agreement with the results of experiments [11] performed with ballistite powders at sufficiently high operating pressures.

On the other hand, a pronounced discrepancy between theory and experiment is to be observed at low pressures. The authors' opinion is that the reason for this discrepancy should be sought in the combustion model employed, in which the chemical reactions and the associated heat releases are concentrated in narrow—almost infinitely thin—zones.

In reality, the reaction zone in the gas, particularly at low pressures, has appreciable dimensions, and the heat release is essentially volumetric in nature [12]. Allowance for this should lead to satisfactory agreement between theory and experiment.

It should be noted that in the present paper, the formula for the acoustic admittance has been derived on the basis of concrete assumptions with regard to the kinetics of a chemical reaction in the k-phase. Recently, Novozhilov proposed a quasi-stationary combustion model for the k-phase, based on the assumption that the steady-state empirical relations $T_S = T_S(p, \varphi)$ and $m = m(p, \varphi)$ are valid in nonsteady-state conditions [13, 14].

In this model, the relations between the disturbances δT_S, δm, $\delta\varphi$, and δp contain the parameters ε, ν, τ, while the parameters μ and z_1 employed in the present paper are replaced by

$$r = (dT_s / dT_0)_p, \qquad \mu^0 = (dT_s / (T_s - T_0) d \ln p)_{T_0}$$

It is readily shown that calculation of the acoustic admittance of a burning surface from Novozhilov's formula leads to the formulas (6.3) and (6.4) if

$$2 - \mu = \mu^0 / (\beta^0 \varepsilon - r\nu), \quad z_1 - 1 = \nu / (\mu^0 \varepsilon - r\nu)$$

are substituted for $2 - \mu$ and $z_1 - 1$, respectively.

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